AP Calculus

Differentiation Rules

<u>Power Rule</u>: $\frac{d}{dx}\left[x^a\right] = a \cdot x^{a-1}$, $a \neq 0$

<u>Constant Rule</u>: $\frac{d}{dx}[c] = 0$

<u>Constant Multiple Rule</u>: $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$

<u>Sum and Difference Rule</u>: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

<u>Product Rule</u>: $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

<u>Quotient Rule</u>: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$, $g(x) \neq 0$

<u>The Chain Rule</u>:

$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

<u>Derivatives of Trigonometric Functions</u>:

$\frac{d}{dx}[\sin u] = \cos u \cdot \frac{du}{dx}$ $\quad$ $\frac{d}{dx}[\tan u] = \sec^2 u \cdot \frac{du}{dx}$ $\quad$ $\frac{d}{dx}[\sec u] = \sec u \tan u \cdot \frac{du}{dx}$

$\frac{d}{dx}[\cos u] = -\sin u \cdot \frac{du}{dx}$ $\quad$ $\frac{d}{dx}[\cot u] = -\csc^2 u \cdot \frac{du}{dx}$ $\quad$ $\frac{d}{dx}[\csc u] = -\csc u \cot u \cdot \frac{du}{dx}$

Derivatives of Inverse Trigonometric Functions:

$$\frac{d}{dx}\left[\sin^{-1} u\right] = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}, \ |\,u\,| < 1$$

$$\frac{d}{dx}\left[\cos^{-1} u\right] = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}, \ |\,u\,| < 1$$

$$\frac{d}{dx}\left[\tan^{-1} u\right] = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left[\cot^{-1} u\right] = -\frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left[\sec^{-1} u\right] = \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}, \ |\,u\,| > 1$$

$$\frac{d}{dx}\left[\csc^{-1} u\right] = -\frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}, \ |\,u\,| > 1$$

Derivatives of Exponential Functions:

$$\frac{d}{dx}\left[e^x\right] = e^x$$

$$\frac{d}{dx}\left[e^u\right] = e^u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left[a^x\right] = a^x \cdot \ln a$$

$$\frac{d}{dx}\left[a^u\right] = a^u \cdot \ln a \cdot \frac{du}{dx}$$

Derivatives of Logarithmic Functions:

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$$

$$\frac{d}{dx}\left[\ln u\right] = \frac{1}{u} \cdot \frac{du}{dx}$$

For $a > 0$ and $a \neq 1$:

$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x \ln a}$$

$$\frac{d}{dx}\left[\log_a u\right] = \frac{1}{u \ln a} \cdot \frac{du}{dx}$$