AP Calculus AB
3.9: P. 170 #1-41 odd, 47, 48, 49, 50, 52, 53

(1) $y = 2e^x$

$\boxed{\dfrac{dy}{dx} = 2e^x}$

(3) $y = e^{-x}$

$\boxed{\dfrac{dy}{dx} = -e^{-x}}$

(5) $y = e^{2x/3}$

$\boxed{\dfrac{dy}{dx} = \dfrac{2}{3}e^{2x/3}}$

(7) $y = xe^2 - e^x$ (remember: e^2 is a constant)

$\boxed{\dfrac{dy}{dx} = e^2 - e^x}$

(9) $y = e^{\sqrt{x}}$

$\dfrac{d}{dx}\left[\sqrt{x}\right] = \dfrac{d}{dx}\left[x^{1/2}\right] = \dfrac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$

$\boxed{\dfrac{dy}{dx} = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}}$

(11) $y = x^{\pi}$

$\boxed{\dfrac{dy}{dx} = \pi x^{\pi - 1}}$

(13) $y = x^{-\sqrt{2}}$

$\boxed{\dfrac{dy}{dx} = -\sqrt{2}\, x^{-\sqrt{2} - 1}}$

(15) $y = 8^x$

$\boxed{\dfrac{dy}{dx} = (\ln 8)8^x}$

(17) $y = 3^{\csc x}$

$\dfrac{dy}{dx} = (-\csc x \cot x)(\ln 3)\, 3^{\csc x}$

$\boxed{\dfrac{dy}{dx} = -(\ln 3)(\csc x \cot x)\, 3^{\csc x}}$

(19) $y = x^{\ln x}$

$\ln y = \ln x^{\ln x}$

$\ln y = (\ln x)(\ln x)$

$\ln y = (\ln x)^2$

$\frac{d}{dx}[\ln y] = \frac{d}{dx}[(\ln x)^2]$

$\frac{1}{y} \cdot \frac{dy}{dx} = 2(\ln x) \cdot \frac{1}{x}$

$\frac{dy}{dx} = y \cdot \frac{2 \ln x}{x}$

$\frac{dy}{dx} = x^{\ln x} \cdot \frac{2 \ln x}{x}$

$$\boxed{\frac{dy}{dx} = 2(\ln x) \cdot x^{(\ln x) - 1} \quad \text{or} \quad \frac{dy}{dx} = \frac{2x^{\ln x} \ln x}{x}}$$

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(21) $y = \ln(x^2)$ D: $x \neq 0$

$y = 2 \ln x$

$$\boxed{\frac{dy}{dx} = \frac{2}{x}, \ x \neq 0}$$

(23) $y = \ln\left(\frac{1}{x}\right)$ D: $x > 0$

$y = \ln x^{-1}$

$y = -\ln x$

$$\boxed{\frac{dy}{dx} = -\frac{1}{x}, \ x > 0}$$

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(25) $y = \ln(x+2)$ D: $x > -2$

$$\boxed{\frac{dy}{dx} = \frac{1}{x+2}, \ x > -2}$$

(27) $y = \ln(2-\cos x)$

$$\frac{dy}{dx} = \frac{1}{2-\cos x} \cdot \sin x$$

$$\boxed{\frac{dy}{dx} = \frac{\sin x}{2-\cos x}}$$

(29) $y = \ln(\ln x)$

$$\frac{dy}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{x \ln x}}$$

(31) $y = \log_4 x^2$ D: $x \neq 0$

$$\frac{dy}{dx} = \frac{2x}{x^2 \ln 4}$$

$$\frac{dy}{dx} = \frac{2}{x \ln 4}$$

$$\frac{dy}{dx} = \frac{2}{x \ln 2^2} = \frac{2}{2x \ln 2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{x \ln 2}, \ x \neq 0}$$

(33) $y = \log_2(3x+1)$ D: $x > -\frac{1}{3}$

$$\boxed{\frac{dy}{dx} = \frac{3}{(3x+1)\ln 2}, \ x > -\frac{1}{3}}$$

(35) $y = \log_2\left(\frac{1}{x}\right)$, $x > 0$

$$\frac{dy}{dx} = \frac{-\frac{1}{x^2}}{\left(\frac{1}{x}\right)\ln 2} \longrightarrow \boxed{\frac{dy}{dx} = -\frac{1}{x \ln 2}, \ x > 0}$$

(37) $y = \ln 2 \cdot \log_2 x$ $D: x > 0$

$$\frac{dy}{dx} = \frac{\ln 2}{x \ln 2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{x}, \ x > 0}$$

(39) $y = \log_{10} e^x$

$$\frac{dy}{dx} = \frac{e^x}{e^x \ln 10}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\ln 10}}$$

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(41) $y = e^x$ tangent to graph, passes through origin

$\frac{dy}{dx} = e^x$ Point of tangency: (x, e^x)

($y = e^x$ does not pass through origin)

tangent line passes through $(0,0)$ and (x, e^x)

$$m_{tan} = \frac{e^x - 0}{x - 0} = \frac{e^x}{x}$$

Solve $\frac{dy}{dx} = e^x = \frac{e^x}{x}$

True only for $x = 1$

When $x = 1$:

$\frac{dy}{dx} = e^1 = e$, passes through $(0,0)$ and $(1, e)$

$$\boxed{\text{Tangent line: } y = ex}$$

43 $y = (\sin x)^x$, $0 < x < \frac{\pi}{2}$ (bonus problem)

$$\ln y = \ln(\sin x)^x$$

$$\ln y = x \ln(\sin x)$$

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[x \ln(\sin x)]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x\left[\frac{1}{\sin x} \cdot \cos x\right] + \ln(\sin x)$$

$$\frac{dy}{dx} = y\left[x \cot x + \ln(\sin x)\right]$$

$$\boxed{\frac{dy}{dx} = (\sin x)^x\left[x \cot x + \ln(\sin x)\right]}$$

(45) $y = \sqrt[5]{\dfrac{(x-3)^4(x^2+1)}{(2x+5)^3}}$ (bonus problem)

$$\ln y = \ln\left[\frac{(x-3)^4(x^2+1)}{(2x+5)^3}\right]^{1/5}$$

$$\ln y = \frac{1}{5}\left[4\ln(x-3) + \ln(x^2+1) - 3\ln(2x+5)\right]$$

$$\frac{d}{dx}\left[\ln y\right] = \frac{1}{5}\frac{d}{dx}\left[4\ln(x-3) + \ln(x^2+1) - 3\ln(2x+5)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{5}\left[\frac{4}{x-3} + \frac{2x}{x^2+1} - \frac{6}{2x+5}\right]$$

$$\frac{dy}{dx} = \frac{1}{5}y\left[\frac{4}{x-3} + \frac{2x}{x^2+1} - \frac{6}{2x+5}\right]$$

$$\boxed{\frac{dy}{dx} = \frac{1}{5}\sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}\left[\frac{4}{x-3} + \frac{2x}{x^2+1} - \frac{6}{2x+5}\right]}$$

(47) $A(t) = 20\left(\frac{1}{2}\right)^{t/140}$

$A'(t) = 20 \cdot \ln\frac{1}{2} \cdot \frac{1}{140} \cdot \left(\frac{1}{2}\right)^{t/140}$

$A'(t) = \frac{1}{7} \cdot \ln\frac{1}{2} \cdot \left(\frac{1}{2}\right)^{t/140}$

$A'(2) = \frac{1}{7} \cdot \ln\frac{1}{2} \cdot \left(\frac{1}{2}\right)^{2/140}$

$A'(2) \approx -0.098$

The plutonium is decaying at approximately 0.098 grams per day when $t = 2$ days.

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(48) a) Chain Rule

$$\frac{d}{dx}\left[\ln(kx)\right] = \frac{1}{kx} \cdot \frac{d}{dx}\left[kx\right] = \frac{1}{kx} \cdot k = \frac{k}{kx} = \frac{1}{x}$$

b) Properties of Logarithms

$$\frac{d}{dx}\left[\ln(kx)\right] = \frac{d}{dx}\left[\ln k + \ln x\right]$$

$$= 0 + \frac{1}{x} = \frac{1}{x}$$

(49). $f(x) = 2^x$

$f'(x) = (\ln 2)(2^x)$

a) $f'(0) = (\ln 2)(2^0) = \ln 2$

b) $f'(0) = \lim_{h \to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{f(h)-1}{h} = \lim_{h \to 0} \frac{2^h - 1}{h}$

c) $\ln 2$ d) $\ln 7$

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(50) (a,b) is on the graph of $y = e^x$ iff.
(b,a) is on the graph of $y = \ln x$

Since there are points (x, e^x) on the graph of $y = e^x$ with arbitrarily large x-coordinates there will be points $(x, \ln x)$ on the graph of $y = \ln x$ with arbitrarily large y-coordinates

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(52) $y = -\frac{1}{2}x^2 + k$ $y = \ln x + c$

$\frac{dy}{dx} = -x$ $\frac{dy}{dx} = \frac{1}{x}$

$(-x)(\frac{1}{x}) = 1$ $\therefore$ at any x, these two curves will have perpendicular tangent lines

53 a) $y = \frac{1}{e}x$ or $y = \frac{x}{e}$

b) The graph of $y = \ln x$ lies below the graph of the line $y = \frac{x}{e}$ for $x > 0$ and $x \neq e$. Therefore, $\ln x < \frac{x}{e}$ for all positive $x \neq e$

c) $\ln x < \frac{x}{e}$

$e(\ln x) < \left(\frac{x}{e}\right)e$

$e \ln x < x$

$\ln(x^e) < x$

d) $\ln(x^e) < x$

$e^{\ln(x^e)} < e^x$

$x^e < e^x$

e) $x^e < e^x$

$\pi^e < e^\pi$

$\boxed{e^\pi \text{ is bigger}}$