<u>AP Calculus AB</u>
<u>3.8</u>: P. 162 #1-17 odd, 21-24 all, 27, 30

(1) $y = \cos^{-1}(x^2)$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$\boxed{\frac{dy}{dx} = -\frac{2x}{\sqrt{1-x^4}}}$$

(3) $y = \sin^{-1}(\sqrt{2}\,t)$

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}} \cdot \sqrt{2}$$

$$\boxed{\frac{dy}{dt} = \frac{\sqrt{2}}{\sqrt{1-2t^2}} = \sqrt{\frac{2}{1-2t^2}}}$$

(5) $y = \sec^{-1}(2s+1)$

$$\frac{dy}{ds} = \frac{1}{|2s+1|\sqrt{(2s+1)^2-1}} \cdot 2$$

$$\frac{dy}{ds} = \frac{2}{|2s+1|\sqrt{4s^2+4s\cancel{+1-1}}}$$

$$\frac{dy}{ds} = \frac{2}{|2s+1| \cdot 2\sqrt{s^2+s}}$$

$$\boxed{\frac{dy}{ds} = \frac{1}{|2s+1|\sqrt{s(s+1)}}}$$

(7) $y = \csc^{-1}(x^2+1),\ x>0$

$$\frac{dy}{dx} = -\frac{1}{|x^2+1|\sqrt{(x^2+1)^2-1}} \cdot 2x$$

$$\frac{dy}{dx} = -\frac{2x}{(x^2+1)\sqrt{x^4+2x^2\cancel{+1-1}}}$$

$$\frac{dy}{dx} = -\frac{2x}{(x^2+1)\,x\sqrt{x^2+2}}$$

$$\boxed{\frac{dy}{dx} = -\frac{2}{(x^2+1)\sqrt{x^2+2}},\ x>0}$$

(9) $y = \sec^{-1}\frac{1}{t},\ 0<t<1$

$y = \cos^{-1} t,\ 0<t<1$

$$\boxed{\frac{dy}{dt} = -\frac{1}{\sqrt{1-t^2}}}$$

(11) $y = \cot^{-1}\sqrt{t}$

$$\frac{dy}{dt} = -\frac{1}{(\sqrt{t})^2+1} \cdot \frac{1}{2}t^{-1/2}$$

$$\boxed{\frac{dy}{dt} = -\frac{1}{2\sqrt{t}\,(t+1)}}$$

(13) $y = s\sqrt{1-s^2} + \cos^{-1}s$

$y = \sqrt{s^2 - s^4} + \cos^{-1}s = (s^2-s^4)^{1/2} + \cos^{-1}s$

$$\frac{dy}{ds} = \frac{1}{2}(s^2-s^4)^{-1/2}(2s-4s^3) + \frac{1}{\sqrt{1-s^2}}$$

$$\frac{dy}{ds} = \frac{s-2s^3}{\sqrt{s^2-s^4}} - \frac{1}{\sqrt{1-s^2}} = \frac{s(1-2s^2)}{s\sqrt{1-s^2}} - \frac{1}{\sqrt{1-s^2}}$$

$$\frac{dy}{ds} = \frac{(1-2s^2)}{\sqrt{1-s^2}} - \frac{1}{\sqrt{1-s^2}} = \frac{1-2s^2-1}{\sqrt{1-s^2}} = \boxed{-\frac{2s^2}{\sqrt{1-s^2}}}$$

(15) $y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1}x, \; x > 1$

$$\frac{dy}{dx} = \frac{1}{1+(\sqrt{x^2-1})^2} \cdot \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x - \frac{1}{|x|\sqrt{x^2-1}}$$

absolute value unnecessary since $x>1$

$$\frac{dy}{dx} = \frac{1}{\cancel{1}+x^2\cancel{-1}} \cdot \frac{x}{\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{dy}{dx} = \frac{x}{x^2\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}} = \frac{1}{x\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}} = \boxed{0}$$

(17) $y = x\sin^{-1}x + \sqrt{1-x^2} = x\sin^{-1}x + (1-x^2)^{1/2}$

$$\frac{dy}{dx} = \left[x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x\right] + \frac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \frac{x}{\sqrt{1-x^2}}$$

$$\boxed{\frac{dy}{dx} = \sin^{-1}x}$$

21 $f(x) = \cos x + 3x$

22 is on next page

a) $f'(x) = 3 - \sin x$

values of $f'(x)$ in $[2,4]$

f is differentiable on $(-\infty, \infty)$

$f'(x)$ is never zero and is always positive, so

f is strictly increasing on $(-\infty, \infty)$

It does not have any horizontal tangents and is one-to-one, so its inverse exists and is differentiable.

b) $f(0) = 1 \qquad f'(0) = 3$

c) $f^{-1}(1) = 0 \qquad (f^{-1})'(1) = \frac{1}{f'(0)}$ since $f^{-1}(1) = 0$

$f(a) = 1$

$a = 0$

$= \frac{1}{3}$

23 $x = \arctan t, \quad t \geq 0$

$$\frac{dx}{dt} = \frac{1}{t^2+1}$$

a) $v(t) = \frac{dx}{dt} = \frac{1}{t^2+1}$ is always positive

$\therefore$ the particle is always moving to the right

b) $a(t) = -\frac{2t}{(t^2+1)^2}$ is negative for $t > 0$

$\therefore$ the particle is always decelerating (since it is always moving to the right)

c) $\lim\limits_{t \to \infty} \arctan t = \frac{\pi}{2}$

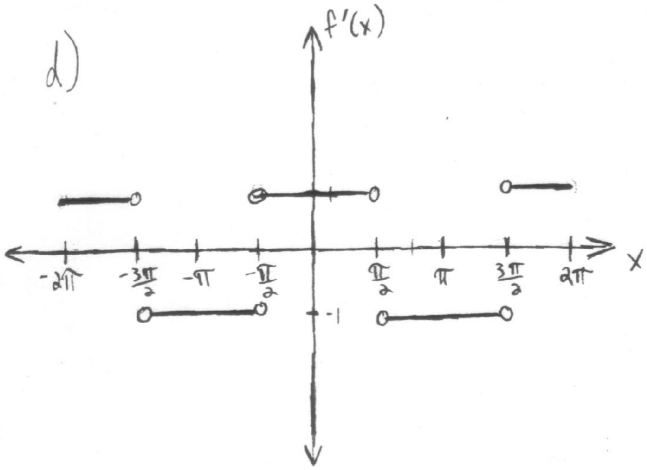
22 $f(x) = \sin^{-1}(\sin x)$ $x: [-2\pi, 2\pi]$ $y: [-4, 4]$

a) D: $\mathbb{R}$ b) R: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ c) $x = \frac{(2n+1)\pi}{2}$, where n is an integer

or

$x = \frac{k\pi}{2}$, where k is an odd integer

d)



e) $f(x) = \sin^{-1}(\sin x)$

$$f'(x) = \frac{1}{\sqrt{1-\sin^2 x}} \cdot \cos x$$

$$f'(x) = \frac{\cos x}{\sqrt{1-\sin^2 x}}$$

$f'(x) = -1$ if $\cos x < 0$ $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

$f'(x) = 1$ if $\cos x > 0$ $\left[-2\pi, -\frac{3\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$

(24) $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

$$\frac{d}{dx}\left[\cos^{-1}x\right] = \frac{d}{dx}\left[\frac{\pi}{2} - \sin^{-1}x\right]$$

$$\frac{d}{dx}\left[\cos^{-1}x\right] = -\frac{1}{\sqrt{1-x^2}}$$

(25) $\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

$$\frac{d}{dx}\left[\cot^{-1}x\right] = \frac{d}{dx}\left[\frac{\pi}{2} - \tan^{-1}x\right]$$

$$\frac{d}{dx}\left[\cot^{-1}x\right] = -\frac{1}{1+x^2}$$

(26) $\csc^{-1}x = \frac{\pi}{2} - \sec^{-1}x$

$$\frac{d}{dx}\left[\csc^{-1}x\right] = \frac{d}{dx}\left[\frac{\pi}{2} - \sec^{-1}x\right]$$

$$\frac{d}{dx}\left[\csc^{-1}x\right] = -\frac{1}{|x|\sqrt{x^2-1}}$$

(27) $y = \tan^{-1}x$

a) right-end behavior model

$\lim_{x\to\infty} \tan^{-1}x = \frac{\pi}{2}$ $\boxed{y = \frac{\pi}{2}}$

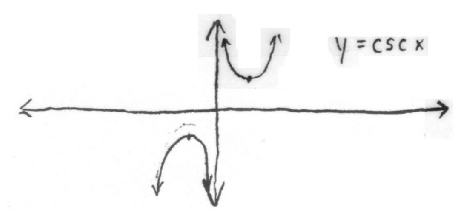
b) left-end behavior model

$\lim_{x\to-\infty} \tan^{-1}x = -\frac{\pi}{2}$ $\boxed{y = -\frac{\pi}{2}}$

c) $\frac{dy}{dx} = \frac{1}{1+x^2} = 0$ no solutions

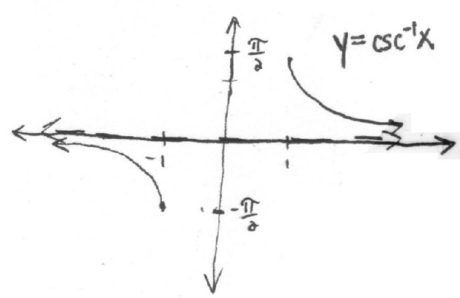
$\boxed{\text{no horizontal tangents}}$

(30) $y = \csc^{-1}x$



a) right-end behavior model

$\lim_{x\to\infty} \csc^{-1}x = 0$ $\boxed{y=0}$



b) left-end behavior model

$\lim_{x\to-\infty} \csc^{-1}x = 0$ $\boxed{y=0}$

c) $\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{1}{|x|\sqrt{x^2-1}} = 0$ $\boxed{\text{no horizontal tangents}}$