AP Calculus AB
3.6: P. 146 #3-69 (multiples of 3), 50, 61, 64

③ $y = \cos(\sqrt{3}x)$

$\frac{dy}{dx} = -\sqrt{3}\sin(\sqrt{3}x)$

⑥ $y = \left(\frac{\sin x}{1+\cos x}\right)^2$

$\frac{dy}{dx} = 2\left(\frac{\sin x}{1+\cos x}\right)\left[\frac{(1+\cos x)(\cos x)-(\sin x)(-\sin x)}{(1+\cos x)^2}\right]$

$\frac{dy}{dx} = \frac{2\sin x}{1+\cos x}\left[\frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}\right]$

$\frac{dy}{dx} = \frac{2\sin x(\cos x+1)}{(1+\cos x)^3} = \frac{2\sin x}{(1+\cos x)^2}$

⑨ $y = (x+\sqrt{x})^{-2}$

$y = (x+x^{1/2})^{-2}$

$\frac{dy}{dx} = -2(x+\sqrt{x})^{-3}\cdot\left(1+\frac{1}{2}x^{-1/2}\right)$

$\frac{dy}{dx} = -2(x+\sqrt{x})^{-3}\left(1+\frac{1}{2\sqrt{x}}\right)$

$\frac{dy}{dx} = \frac{-2\left(1+\frac{1}{2\sqrt{x}}\right)}{(x+\sqrt{x})^3} = \frac{-2-\frac{1}{\sqrt{x}}}{(x+\sqrt{x})^3} = \frac{-2\sqrt{x}-1}{\sqrt{x}(x+\sqrt{x})^3}$

⑫ $y = x^3(2x-5)^4$

$\frac{dy}{dx} = x^3\left[4(2x-5)^3(2)\right] + 3x^2(2x-5)^4$

$\frac{dy}{dx} = 8x^3(2x-5)^3+3x^2(2x-5)^4$

$\frac{dy}{dx} = x^2(2x-5)^3\left[8x+3(2x-5)\right]$

$\frac{dy}{dx} = x^2(14x-15)(2x-5)^3$

⑮ $y = \frac{3}{\sqrt{2x+1}}$

$y = 3(2x+1)^{-1/2}$

$\frac{dy}{dx} = -\frac{3}{\cancel{2}}(2x+1)^{-3/2}(\cancel{2})$

$\frac{dy}{dx} = \frac{-3}{(2x+1)^{3/2}}$

$\frac{dy}{dx} = \frac{-3}{\sqrt{(2x+1)^3}}$

$\frac{dy}{dx} = \frac{-3}{(2x+1)\sqrt{2x+1}}$

(18) $y = (1+\cos 2x)^2$

$$\frac{dy}{dx} = 2(1+\cos 2x)\left[-2\sin 2x\right] = -4(\sin 2x)(1+\cos 2x)$$

(21) $s = \cos\left(\frac{\pi}{2} - 3t\right)$

$$\frac{ds}{dt} = -\sin\left(\frac{\pi}{2}-3t\right)\cdot(-3) = 3\sin\left(\frac{\pi}{2}-3t\right)$$

(24) $s = \sin\left(\frac{3\pi}{2}t\right) + \cos\left(\frac{7\pi}{4}t\right)$

$$\frac{ds}{dt} = \frac{3\pi}{2}\cos\left(\frac{3\pi}{2}t\right) - \frac{7\pi}{4}\sin\left(\frac{7\pi}{4}t\right)$$

(27) $r = \sqrt{\theta\sin\theta} = (\theta\sin\theta)^{1/2}$

$$\frac{dr}{d\theta} = \frac{1}{2}(\theta\sin\theta)^{-1/2}\left[\theta\cos\theta + \sin\theta\right]$$

$$\frac{dr}{d\theta} = \frac{\theta\cos\theta + \sin\theta}{2\sqrt{\theta\sin\theta}}$$

(33) $f(u) = u^5+1$ $\quad g(x) = \sqrt{x}$

$f'(u) = 5u^4$ $\quad g'(x) = \frac{1}{2\sqrt{x}}$

$$(f\circ g)'(1) = f'(g(1))\cdot g'(1)$$
$$= f'(1)\cdot\frac{1}{2}$$
$$= 5\cdot\frac{1}{2} = \boxed{\frac{5}{2}}$$

(30) $y = \cot x$

$y' = -\csc^2 x$

$y'' = -2\csc x\left[\csc x\cot x\right]$

$y'' = -2\csc^2 x\cot x$

(36) $f(u) = u + \frac{1}{\cos^2 u}$, $u = g(x) = \pi x$, $x = \frac{1}{4}$

$f(u) = u + (\cos u)^{-2}$ $\qquad g'(x) = \pi$

$f'(u) = 1 - 2(\cos u)^{-3}(-\sin u)$

$f'(u) = 1 + \frac{2\sin u}{\cos^3 u}$

$$(f \circ g)'\left(\tfrac{1}{4}\right) = f'(g(1/4)) \cdot g'\left(\tfrac{1}{4}\right)$$

$$= f'\left(\tfrac{\pi}{4}\right) \cdot (\pi)$$

$$= \pi\left[1 + \frac{2\sin\frac{\pi}{4}}{\cos^3\frac{\pi}{4}}\right] = \pi\left[1 + \frac{2\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)^3}\right]$$

$$= \pi\left[1 + \frac{\sqrt{2}}{\frac{2\sqrt{2}}{8}}\right] = \pi\left[1 + \frac{1}{\frac{2}{8}}\right] = \boxed{5\pi}$$

(39) $y = \cos(6x+2)$

a) $y = \cos u \quad u = 6x+2$

$\frac{dy}{du} = -\sin u \quad \frac{du}{dx} = 6$

$\frac{dy}{dx} = -6\sin u = -6\sin(6x+2)$

b) $y = \cos 2u \quad u = 3x+1$

$\frac{dy}{du} = -2\sin 2u \quad \frac{du}{dx} = 3$

$\frac{dy}{dx} = 3(-2\sin 2u) = -6\sin[2(3x+1)]$

$= -6\sin(6x+2)$

(42) $x = \sin 2\pi t \ , \ y = \cos 2\pi t \ , \ t = -\frac{1}{6}$

$\frac{dx}{dt} = 2\pi \cos 2\pi t \quad \frac{dy}{dt} = -2\pi \sin 2\pi t \qquad \left(\sin\left(-\frac{\pi}{3}\right), \cos\left(-\frac{\pi}{3}\right)\right)$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{dy}{dx} = \frac{-2\pi \sin 2\pi t}{2\pi \cos 2\pi t} = \frac{-\sin 2\pi t}{\cos 2\pi t} = -\tan 2\pi t$

At $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $t = -\frac{1}{6} \rightarrow \frac{dy}{dx} = -\tan\left(-\frac{\pi}{3}\right) = \sqrt{3}$

$y = mx + b$

$\frac{1}{2} = \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) + b \qquad \boxed{y = \sqrt{3}x + 2}$

$\frac{1}{2} = -\frac{3}{2} + b \qquad b = 2$

(45) $x = t \ , \ y = \sqrt{t} \ , \ t = 1/4$

$\frac{dx}{dt} = 1 \quad \frac{dy}{dt} = \frac{1}{2\sqrt{t}} \qquad \left(\frac{1}{4}, \frac{1}{2}\right)$

$\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{t}}}{1} = \frac{1}{2\sqrt{t}}$

For $t = \frac{1}{4}$ at $\left(\frac{1}{4}, \frac{1}{2}\right) \rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{1}{4}}} = 1$

$y = mx + b$

$\frac{1}{2} = 1\left(\frac{1}{4}\right) + b \qquad \boxed{y = x + \frac{1}{4}}$

$b = \frac{1}{4}$

(48) $x = \cos t$, $y = 1 + \sin t$, $t = \frac{\pi}{2}$

$\frac{dx}{dt} = -\sin t$ $\frac{dy}{dt} = \cos t$ $(\cos\frac{\pi}{2}, 1 + \sin\frac{\pi}{2})$

$(0, 2)$

$$\frac{dy}{dx} = \frac{\cos t}{-\sin t} = -\cot t$$

At $t = \frac{\pi}{2}$, $(0, 2) \to \frac{dy}{dx} = -\cot\frac{\pi}{2} = 0$

$\boxed{y = 2}$

(51) $s = \cos\theta$ $\frac{ds}{d\theta} = -\sin\theta$ [$\frac{d\theta}{dt} = 5$ was given]

$$\frac{ds}{dt} = \frac{ds}{d\theta} \cdot \frac{d\theta}{dt} = -5\sin\theta$$

When $\theta = \frac{3\pi}{2}$, $\frac{ds}{dt} = -5\sin\frac{3\pi}{2} = \boxed{5}$

(54) $y = \sin mx$

$$\frac{dy}{dx} = m\cos mx$$

At $(0,0)$, $\frac{dy}{dx} = m\cos 0 = m$

$\boxed{y = mx}$

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a) $\frac{d}{dx}\left[5f(x) - g(x)\right] = 5f'(x) - g'(x)$

At $x=1$: $5\left(-\frac{1}{3}\right) - \left(-\frac{8}{3}\right) = \boxed{1}$

b) $\frac{d}{dx}\left[f(x)\, g^3(x)\right] = f(x) \cdot 3g^2(x) \cdot g'(x) + f'(x) g^3(x)$

At $x=0$: $(1)(3)(1^2)\left(\frac{1}{3}\right) + (5)(1^3) = \boxed{6}$

c) $\frac{d}{dx}\left[\frac{f(x)}{g(x)+1}\right] = \frac{\left[g(x)+1\right] \cdot f'(x) - f(x)g'(x)}{\left[g(x)+1\right]^2}$

At $x=1$: $\frac{\left[-4+1\right]\left(-\frac{1}{3}\right) - 3\left(-\frac{8}{3}\right)}{(-4+1)^2} = \frac{1+8}{9} = \boxed{1}$

d) $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$

At $x=0$: $f'(1) \cdot \left(\frac{1}{3}\right) = \left(-\frac{1}{3}\right)\left(\frac{1}{3}\right) = \boxed{-\frac{1}{9}}$

e) $\frac{d}{dx}\left[g(f(x))\right] = g'(f(x)) \cdot f'(x)$

At $x=0$: $g'(1) \cdot (5) = \left(-\frac{8}{3}\right)(5) = \boxed{-\frac{40}{3}}$

f) $\frac{d}{dx}\left[(g(x)+f(x))^{-2}\right] = -2(g(x)+f(x))^{-3}\left[g'(x)+f'(x)\right]$

At $x=1$: $-2(-4+3)^{-3}\left[-\frac{8}{3} - \frac{1}{3}\right] = \boxed{-6}$

g) $\frac{d}{dx}\left[f(x+g(x))\right] = f'(x+g(x))\left[1+g'(x)\right]$

At $x=0$: $f'(1)\left[1+\frac{1}{3}\right] = \left(-\frac{1}{3}\right)\left(\frac{4}{3}\right) = \boxed{-\frac{4}{9}}$

(60) $s(t) = A\cos(2\pi bt)$

$v(t) = -2\pi bA\sin(2\pi bt)$

$a(t) = -4\pi^2 b^2 A\cos(2\pi bt)$

$j(t) = 8\pi^3 b^3 A\sin(2\pi bt)$

Doubling the frequency of the position will:

- double the amplitude of the velocity
- quadruple the amplitude of the acceleration
- octuple the amplitude of the jerk

(63) $v(s) = k\sqrt{s} = ks^{1/2}$

acceleration $= \frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt}$

$\frac{dv}{dt} = \frac{k}{2\sqrt{s}}\cdot\frac{ds}{dt}$

$\frac{dv}{dt} = \frac{k}{2\sqrt{s}}\cdot v = \frac{k}{2\sqrt{s}}\cdot k\sqrt{s}$

$\frac{dv}{dt} = \frac{k^2}{2}$, which is constant since k is constant

(66) $T = 2\pi\sqrt{\frac{L}{g}}$ $\qquad \frac{dL}{du} = kL$

$\frac{dT}{du} = \frac{dT}{dL}\cdot\frac{dL}{du}$

$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\left(\frac{L}{g}\right)^{1/2}$

$\frac{dT}{dL} = \pi\left(\frac{L}{g}\right)^{-1/2}\cdot\frac{1}{g} = \frac{\pi\sqrt{g}}{g\sqrt{L}}$

$\frac{dT}{du} = \frac{\pi\sqrt{g}}{g\sqrt{L}}\cdot kL = \frac{\pi kL\sqrt{g}}{g\sqrt{L}}$

$\frac{dT}{du} = \frac{\pi k\sqrt{L}}{\sqrt{g}} = \pi k\sqrt{\frac{L}{g}}$

$\pi\sqrt{\frac{L}{g}} = \frac{1}{2}T = \frac{T}{2}$, so

$\frac{dT}{du} = \frac{kT}{2}$

(69) As $h \to 0$, the graph of $y = \frac{\sin 2(x+h) - \sin 2x}{h}$ approaches the graph of $y = 2\cos 2x$.

This is because $y = 2\cos 2x$ is the derivative of $y = \sin 2x$ and $y = \frac{\sin 2(x+h) - \sin 2x}{h}$ is the difference quotient used to define the derivative of $y = \sin 2x$

(50) radius passes through $(0,0)$ & $(2\cos t, 2\sin t)$

Slope of the radius $= \dfrac{2\sin t - 0}{2\cos t - 0} = \tan t$

$x = 2\cos t \qquad y = 2\sin t$

$\dfrac{dx}{dt} = -2\sin t \qquad \dfrac{dy}{dt} = 2\cos t$

$\dfrac{dy}{dx} = \dfrac{2\cos t}{-2\sin t} = -\cot t$

Slope of radius: $\tan t$

Slope of tangent line: $-\cot t$

$(\tan t)(-\cot t) = -1 \quad \therefore$ the radius & tangent line are perpendicular

(61) $y = 37\sin\left[\frac{2\pi}{365}(x-101)\right] + 25$

a) $\dfrac{dy}{dx} = \dfrac{74\pi}{365}\cos\left[\frac{2\pi}{365}(x-101)\right]$

This will be maximized when

$\frac{2\pi}{365}(x-101) = 0$

$x = 101$

101^{st} day of the year is April 11

b) $y'(101) = \dfrac{74\pi}{365}\cos\left[\frac{2\pi}{365}(101-101)\right]$

$= \dfrac{74\pi}{365} \approx 0.637$ degrees per day

(64) $v = \dfrac{k}{\sqrt{s}} = ks^{-1/2}$

$\dfrac{dv}{dt} = \dfrac{dv}{ds} \cdot \dfrac{ds}{dt}$

$\dfrac{dV}{dt} = -\dfrac{k}{2\sqrt{s^3}} \cdot V$

$\dfrac{dV}{dt} = -\dfrac{k}{2\sqrt{s^3}} \cdot \dfrac{k}{\sqrt{s}}$

$\dfrac{dV}{dt} = -\dfrac{k^2}{2\sqrt{s^4}} =$

$\dfrac{dV}{dt} = -\dfrac{k}{2s^2}, \; s \geq 0$