AP Calculus AB
3.3: P. 120 #1-33 odd, 28, 30, 34

(1) $y = -x^2 + 3$

$\frac{dy}{dx} = -2x$

$\frac{d^2y}{dx^2} = -2$

(3) $y = 2x + 1$

$\frac{dy}{dx} = 2$

$\frac{d^2y}{dx^2} = 0$

(5) $y = \frac{x^3}{3} + \frac{x^2}{2} + x$

$\frac{dy}{dx} = x^2 + x + 1$

$\frac{d^2y}{dx^2} = 2x + 1$

(7) $y = x^4 - 7x^3 + 2x^2 + 15$

$\frac{dy}{dx} = 4x^3 - 21x^2 + 4x = x(4x^2 - 21x + 4)$

$\frac{d^2y}{dx^2} = 12x^2 - 42x + 4 = 2(6x^2 - 21x + 2)$

(9) $y = 4x^{-2} - 8x + 1$

$\frac{dy}{dx} = -8x^{-3} - 8$

$= -8(x^{-3} + 1)$

$= -8\left(\frac{1}{x^3} + 1\right)$

$\frac{d^2y}{dx^2} = -8(-3x^{-4}) = \frac{24}{x^4}$

(11) $y = (x+1)(x^2+1)$

a) $\frac{dy}{dx} = (x+1)(2x) + (x^2+1)(1)$

$\frac{dy}{dx} = 2x^2 + 2x + x^2 + 1$

$\frac{dy}{dx} = 3x^2 + 2x + 1$

b) $y = x^3 + x^2 + x + 1$

$\frac{dy}{dx} = 3x^2 + 2x + 1$

(13) $y = \frac{2x+5}{3x-2}$

$$\frac{dy}{dx} = \frac{(3x-2)(2) - (2x+5)(3)}{(3x-2)^2} = \frac{6x-4-6x-15}{(3x-2)^2} = -\frac{19}{(3x-2)^2}$$

(15) $$y = \frac{(x-1)(x^2+x+1)}{x^3} = \frac{x^3 - x^2 + x^2 - x + x - 1}{x^3} = \frac{x^3-1}{x^3}$$

$$y = 1 - \frac{1}{x^3} = 1 - x^{-3} \qquad \frac{dy}{dx} = 3x^{-4} = \frac{3}{x^4}$$

(17) $$y = \frac{x^2}{1-x^3}$$

$$\frac{dy}{dx} = \frac{(1-x^3)(2x) - (x^2)(-3x^2)}{(1-x^3)^2} = \frac{2x - 2x^4 + 3x^4}{(1-x^3)^2} = \frac{2x+x^4}{(1-x^3)^2}$$

$$\frac{dy}{dx} = \frac{x(2+x^3)}{(1-x^3)^2}$$

(19) $$y = \frac{(x+1)(x+2)}{(x-1)(x-2)} = \frac{x^2+3x+2}{x^2-3x+2}$$

$$\frac{dy}{dx} = \frac{(x^2-3x+2)(2x+3) - (x^2+3x+2)(2x-3)}{(x^2-3x+2)^2}$$

$$\frac{dy}{dx} = \frac{2x^3 + 3x^2 - 6x^2 - 9x + 4x + 6 - 2x^3 + 3x^2 - 6x^2 + 9x - 4x + 6}{[(x-1)(x-2)]^2}$$

$$\frac{dy}{dx} = \frac{-6x^2+12}{[(x-1)(x-2)]^2} = \frac{-6(x^2-2)}{[(x-1)(x-2)]^2}$$

(21) $y = c \cdot f(x)$

$\frac{dy}{dx} = c \cdot \frac{d}{dx}[f(x)] + f(x) \cdot \frac{d}{dx}[c]$

$\frac{dy}{dx} = c \cdot \frac{d}{dx}[f(x)] + f(x) \cdot 0$

$\frac{dy}{dx} = c \cdot \frac{d}{dx}[f(x)]$

(23) a) $\frac{d}{dx}[uv] = uv' + vu' = (5)(2) + (-1)(-3) = 13$

b) $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2} = \frac{(-1)(-3) - (5)(2)}{(-1)^2} = -7$

c) $\frac{d}{dx}\left[\frac{v}{u}\right] = \frac{uv' - vu'}{u^2} = \frac{(5)(2) - (-1)(-3)}{5^2} = \frac{7}{25}$

d) $\frac{d}{dx}(7v - 2u) = 7v' - 2u' = 7(2) - 2(-3) = 20$

(25) $\frac{dy}{dx} = 2x + 5$ at $x = 3$, $\frac{dy}{dx} = 11$ iii

(27) $\frac{dy}{dx} = 3x^2 - 3$ at $(2, 3)$ $\frac{dy}{dx} = 9$

normal line: $m = -\frac{1}{9}$

$y = mx + b$

$3 = -\frac{1}{9}(2) + b$

$\frac{29}{9} = -\frac{2}{9} + b$

$b = \frac{29}{9}$

$y = -\frac{1}{9}x + \frac{29}{9}$

(29) $\frac{dy}{dx} = 6x^2 - 6x - 12$ $\qquad y = 2x^3 - 3x^2 - 12x + 20$

$6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$(2, 0)$

$(-1, 27)$

(31) $y = \frac{4x}{x^2+1}$

$\frac{dy}{dx} = \frac{(x^2+1)(4) - (4x)(2x)}{(x^2+1)^2} = \frac{4x^2+4-8x^2}{(x^2+1)^2} = \frac{4-4x^2}{(x^2+1)^2} = \frac{4(1-x^2)}{(x^2+1)^2}$

At $(0,0)$:

$\frac{dy}{dx} = 4$ tangent line is $y = 4x$

At $(1,2)$:

$\frac{dy}{dx} = 0$ tangent line is $y = 2$

(33) $P = \frac{nRT}{V-nb} - \frac{an^2}{V^2} = \frac{nRT}{V-nb} - an^2V^{-2}$

$\frac{dP}{dV} = \frac{(V-nb)\frac{d}{dV}(nRT) - (nRT)\frac{d}{dV}(V-nb)}{(V-nb)^2} + 2an^2V^{-3}$

$\frac{dP}{dV} = \frac{0 \cdot \frac{d}{dV}(nRT) - (nRT)(1)}{(V-nb)^2} + 2an^2V^{-3}$

$\frac{dP}{dV} = \frac{-nRT}{(V-nb)^2} + \frac{2an^2}{V^3} = \frac{2an^2}{V^3} - \frac{nRT}{(V-nb)^2}$

(28) $y = x^3 + x$

$\frac{dy}{dx} = 3x^2 + 1$

$3x^2 + 1 = 4$

$3x^2 = 3$

$x = \pm 1$

$(1, 2)$ $(-1, -2)$

Tangent at $(1,2)$:

$y = mx + b$

$2 = 4(1) + b$

$b = -2$

$y = 4x - 2$

Tangent at $(-1,-2)$:

$y = mx + b$

$-2 = 4(-1) + b$

$-2 = -4 + b$

$b = 2$

$y = 4x + 2$

The smallest slope of this curve is 1.

$3x^2 + 1 = 1$

$3x^2 = 0$

$x = 0$

The curve has this slope at $x = 0$.

(30) $y = x^3$

$\frac{dy}{dx} = 3x^2$

at $(-2, -8)$ $\frac{dy}{dx} = 12$

$y = mx + b$

$-8 = 12(-2) + b$

$-8 = -24 + b$

$b = 16$

$y = 12x + 16$

x-intercept: $(-\frac{4}{3}, 0)$

y-intercept: $(0, 16)$

(34) $s = 4.9t^2$

$\frac{ds}{dt} = 9.8t$

$\frac{d^2s}{dt^2} = 9.8$