AP Calculus AB
3.2: P. 111 #1-17 odd, 18-23 all, 29, 31

(1) left-hand derivative: 0
right-hand derivative: 1

(3) left-hand derivative: $\frac{1}{2}$
right-hand derivative: 2

(5) a) $[-3,2]$
b) none
c) none

(7) a) $[-3,0)\cup(0,3]$
b) none
c) $x=0$

(9) a) $[-1,0)\cup(0,2]$
b) $x=0$
c) none

(11) discontinuity

(13) corner

(15) corner

(17) $f(x)=\dfrac{x^3-8}{x^2-4x-5}=\dfrac{(x-2)(x^2+2x+4)}{(x+1)(x-5)}$

non-differentiable at $x=-1$ or $x=5$

(18) $h(x)=\sqrt[3]{3x-6}+5$
not differentiable at $x=2$

(19) $P(x)=\sin(|x|)-1$
not differentiable at $x=0$

(20) $Q(x)=3\cos(|x|)$
differentiable at all x-values

(21) $g(x) = \begin{cases} (x+1)^2, & x \le 0 \\ 2x+1, & 0 < x < 3 \\ (4-x)^2, & x \ge 3 \end{cases}$ $g(x) = \begin{cases} x^2+2x+1, & x \le 0 \\ 2x+1, & 0 < x < 3 \\ 16-8x+x^2, & x \ge 3 \end{cases}$

$g'(x) = \begin{cases} 2x+2, & x \le 0 \\ 2, & 0 < x < 3 \\ 2x-8, & x \ge 3 \end{cases}$

#22 is on next page

not differentiable at $x=3$

differentiable on $(-\infty, 3) \cup (3, \infty)$

(23) a) $x=0$ is not in the domain of either function

b) Why bother? We know they don't exist

(29) a) $a+b=2$

b) For the function to be continuous:

$a+b=2$ and $f(1)=2$

Derivative from the left at $x=1$ is -1

Derivative from the right at $x=1$ is $2a+b$

To be differentiable, the two must be equal

$2a+b=-1$

$2a+(2-a)=-1$

$a+2=-1$

$\boxed{\begin{array}{l} a=-3 \\ b=5 \end{array}}$

(22) $C(x) = x|x|$

$$C(x) = \begin{cases} x(-x), & x<0 \\ x(x), & x>0 \end{cases} \qquad C(x) = \begin{cases} -x^2, & x<0 \\ x^2, & x>0 \end{cases}$$

$$C'(x) = \begin{cases} -2x, & x<0 \\ 2x, & x>0 \end{cases}$$

Possible non-differentiability at $x=0$

Derivative from the left of $x=0$: 0
Derivative from the right of $x=0$: 0
} differentiable at $x=0$

Differentiable on $(-\infty, \infty)$

(31) a) $f(0) = 0$

$\lim_{x\to 0^-} x^2 \sin\frac{1}{x} = 0 \qquad \lim_{x\to 0^+} x^2 \sin\frac{1}{x} = 0$

Therefore, f is continuous at $x=0$.

b) $$\frac{f(0+h)-f(0)}{h} = \frac{(0+h)\sin\frac{1}{0+h} - 0}{h} = \frac{h\sin\frac{1}{h}}{h} = \sin\frac{1}{h}$$

c) $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$ does not exist due to infinitely oscillating behavior in the interval $[-1, 1]$ as h approaches 0.

d) No.

e) $$\lim_{h\to 0} \frac{g(0+h)-g(0)}{h} = \lim_{h\to 0} \frac{h^2 \sin\frac{1}{h} - 0}{h} = \lim_{h\to 0} h\sin\frac{1}{h} = 0$$

$\therefore g'(0) = 0$