**AP Calculus AB**
**3.1:** P. 101 #1-6 all, 7-25 odd, 26

(1) a) tangent line $f(2)=3 \quad f'(2)=5$

$y=mx+b$
$3=5(2)+b$
$b=-7$

$\boxed{y=5x-7}$

b) normal line $f(2)=3 \quad m=-\frac{1}{5}$

$y=mx+b$
$3=-\frac{1}{5}(2)+b$
$\frac{15}{5}=-\frac{2}{5}+b$
$b=\frac{17}{5}$

$\boxed{y=-\frac{1}{5}x+\frac{17}{5}}$

(2) $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

$f'(3)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h}$

$f'(3)=\lim_{h\to 0}\frac{\frac{3-(3+h)}{3(3+h)}}{h}=\lim_{h\to 0}\frac{\frac{-h}{3(3+h)}}{h}=\lim_{h\to 0}-\frac{1}{3(3+h)}=-\frac{1}{3(3+0)}$

$\boxed{f'(3)=-\frac{1}{9}}$

(3) $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

$f'(3)=\lim_{x\to 3}\frac{\frac{1}{x}-\frac{1}{3}}{x-3}=\lim_{x\to 3}\frac{\frac{3-x}{3x}}{x-3}=\lim_{x\to 3}\frac{\frac{-x+3}{3x}}{x-3}$

$f'(3)=\lim_{x\to 3}\frac{\frac{-(x-3)}{3x}}{(x-3)}=\lim_{x\to 3}-\frac{1}{3x}=\boxed{-\frac{1}{9}}$

(4) $f(x)=3x-12$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{[3(x+h)-12]-[3x-12]}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\cancel{3x}+3h-\cancel{12}-\cancel{3x}+\cancel{12}}{h}=\lim_{h\to 0}\frac{3h}{h}=\lim_{h\to 0}3=\boxed{3}$$

(5) $y=7x$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{7(x+h)-7x}{h}=\lim_{h\to 0}\frac{7x+7h-7x}{h}$$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{7h}{h}=\lim_{h\to 0}7=\boxed{7}$$

(6) $y=x^2$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}$$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{2xh+h^2}{h}=\lim_{h\to 0}(2x+h)=\boxed{2x}$$

(7) b

(9) d

(11) $y=2x^2-13x+5$

$\frac{dy}{dx}=4x-13$ $(3,-16)$

At $x=3$, $\frac{dy}{dx}=-1$

$y=mx+b$

$-16=-1(3)+b$

$b=-13$

$\boxed{y=-x-13}$

(13) ii

(15) a) increasing fastest approximately $\frac{1}{4}$ through the year

b) yes: end/beginning of year & middle of year

c) positive: first half of year
negative: second half of year

(17) a) largest: derivative is zero
Smallest: derivative is zero

b) when derivative is largest, the population is 1700
when derivative is smallest, the population is ~1300

(19) a) derivative is the change in position over time, which is velocity - skier's downhill velocity

b) feet/second

c) To graph the derivative:

| Midpoint of time segment | Slope |
|---|---|
| 0.5 | 3.3 |
| 1.5 | 10 |
| 2.5 | 16.6 |
| 3.5 | 23.3 |
| 4.5 | 30 |
| 5.5 | 36.6 |
| 6.5 | 43.2 |
| 7.5 | 49.9 |
| 8.5 | 56.6 |
| 9.5 | 63.2 |

Use TI-83 or TI-89 to make a scatter plot of the data. Data appears linear. Use linear regression to find the line of best fit.

Equation of line of best fit is the equation of the derivative, which is approximately:

$s'(t)=v(t)=6.65t$

(21) $y = \cos x$ could be the derivative of $y = \sin x$

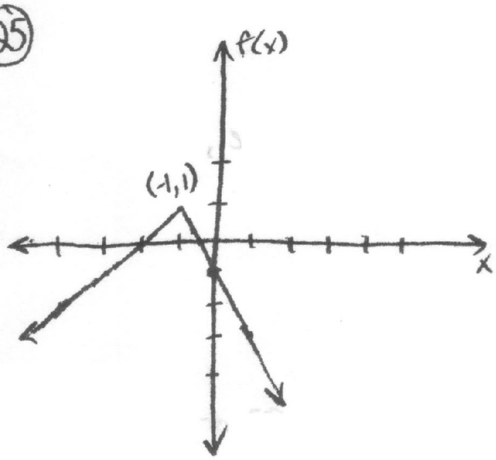
(23) $y = \sqrt{x} = x^{1/2}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-1/2} = -\frac{1}{2\sqrt{x}}$

at $x = 0$: $\frac{dy}{dx}$ is undefined

Therefore, $y = \sqrt{x}$ does not have a right-hand derivative at $x = 0$.

(25)




(26) $f(x) = \begin{cases} x^2, & x \le 1 \\ 2x, & x > 1 \end{cases}$

a) $2x$

b) $2$

c) $2$

d) $2$

e) yes, because the one-sided limit at $x = 1$ exists

f) $2$

g) does not exist

h) no, because the right-hand derivative does not exist